
Student Number

2021 Glenwood High School
Year 12 – Trial HSC Examination

Mathematics Extension 1

General Instructions

* Reading Time – 10 minutes
* Working time – 2 hours
* Write using black pen
* NESA approved calculators may be used
* A reference sheet is provided
* For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2 – 5)

* Attempt Questions 1-10
* Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 11)

* Attempt Questions 11 – 14
* Allow about 1 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1. What is the value of $\cos(75°)$?

A. $\frac{\sqrt{3}-1}{2\sqrt{2}}$

B. $\frac{\sqrt{6}-\sqrt{2}}{2}$

C. $\frac{\sqrt{3}+1}{2\sqrt{2}}$

D. $\frac{\sqrt{6}+\sqrt{2}}{2}$

2. The vectors $4\underset{\sim}{i} - \underset{\sim}{j}$ and $3\underset{\sim}{i} + x\underset{\sim}{j}$ are perpendicular. What is the value of x?

A. –12

B. –1

C. 3

D. 12

3. The probability of Hannah scoring a goal in a netball game is 0.45. What is the least number of games that must be played to ensure the probability of scoring a goal at least once is more than 0.9?

A. 2

B. 3

C. 4

D. 5

4. What is the domain and range of the function $f(x) = 2\ \sin^{-1} 3x$?

A. $-\frac{1}{3} \leq x \leq \frac{1}{3}, -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

B. $-\frac{1}{3} \leq x \leq \frac{1}{3}, -\pi \leq y \leq \pi$

C. $-3 \leq x \leq 3\ , -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

D. $-3 \leq x \leq 3\,, -2 \leq y \leq 2$

5. A sock drawer has 8 black socks and 14 white socks.

What is the minimum number of times someone needs to remove socks to ensure that two different coloured socks are selected?

A. 2

B. 14

C. 15

D. 22

6. The variance of a Bernoulli distribution is $\frac{3}{16}$.

What is a possible value for the mean of the distribution?

A. $\frac{3}{16}$

B. $\frac{5}{16}$

C. $\frac{3}{4}$

D. $\frac{5}{4}$

7. A particle is initially positioned at $x = -2$. Its motion has velocity of $v = \dfrac{1}{t+e}$.

Which of the following will be true when the particle reaches the origin?

A. $t = 2, v = \dfrac{1}{2+e}$

B. $t = e, \dot{x} = \dfrac{1}{2e}$

C. $t = e^2, v = \dfrac{1}{e^2 + e}$

D. $t = e^3 - e, v = e^{-3}$

8. It is known that $\sin x = \frac{1}{3}$, where $\frac{\pi}{2} < x < \pi$.

What is the value of $\cos 2x$?

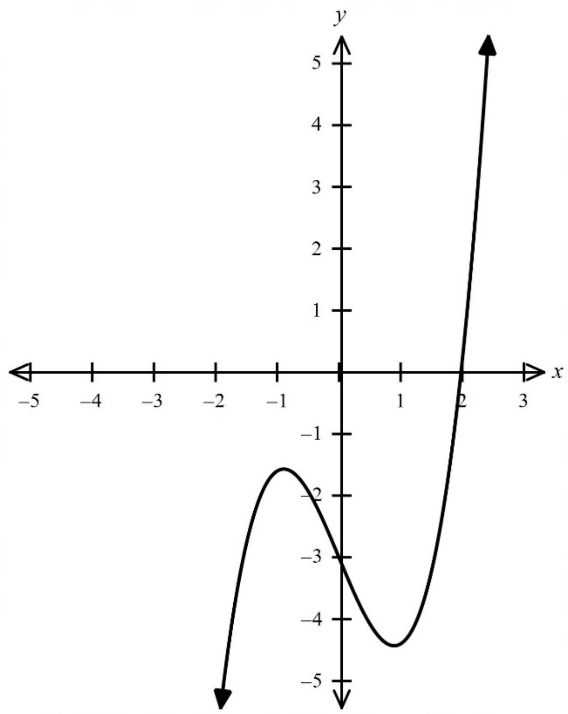
A. $-\frac{7}{9}$

B. $\frac{7}{9}$

C. $\frac{2}{3}$

D. $\frac{-4\sqrt{2}}{3}$

9. Consider the graph of $y = f(x)$ shown here:



Which one of the following would have 2 more roots than $f(x)$?

A. $y = -2 \times f(x)$

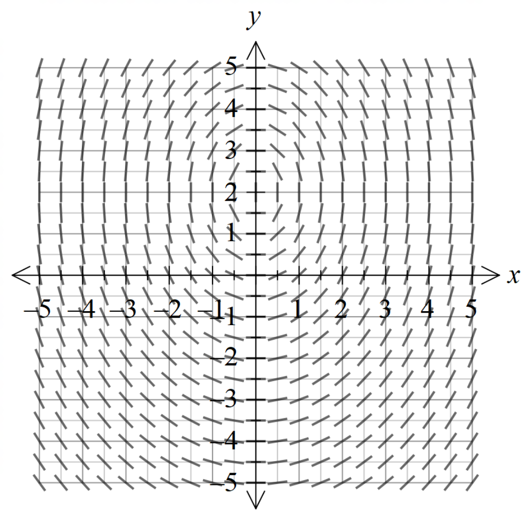
B. $y = f(x) + 3$

C. $y = f^{-1}(x)$

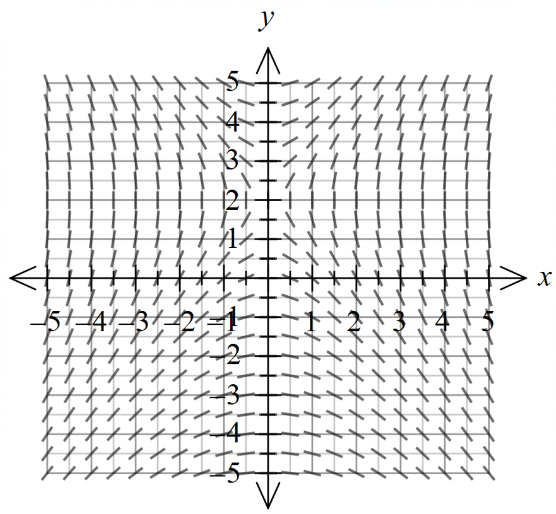
D. $y = f(x + 3)$

10. Which of the following best represents the direction field for the differential equation $\frac{dy}{dx} = \frac{-2x}{y-2}$?

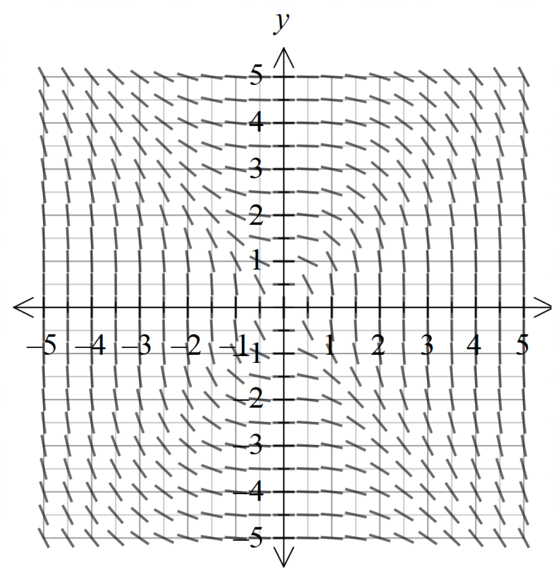
A.



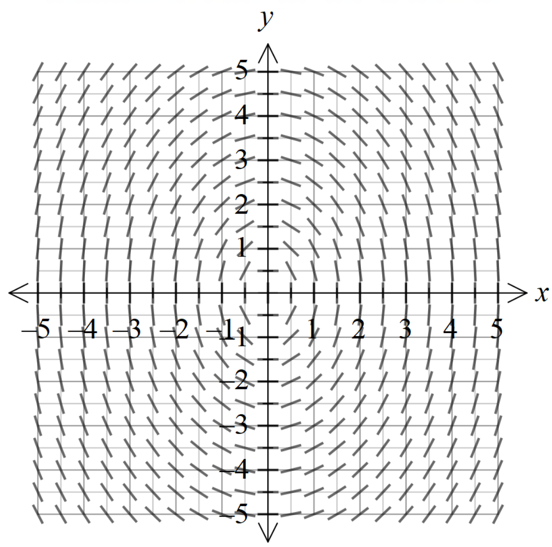
B.



C.



D.



Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

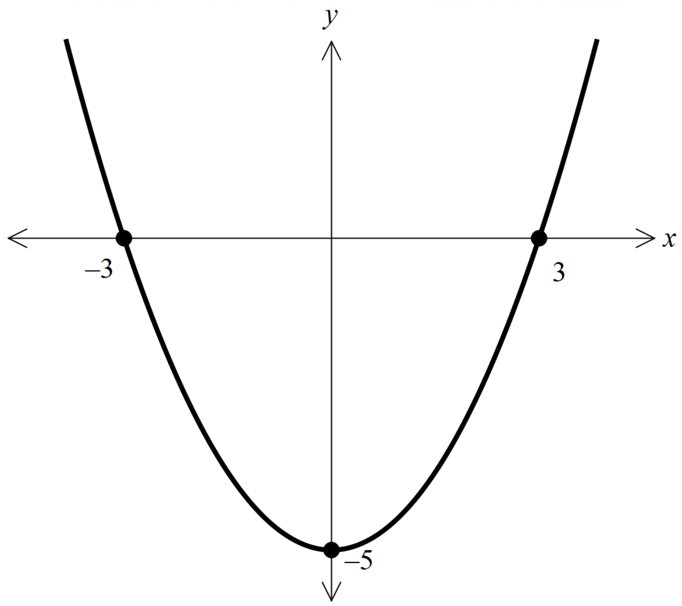
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use a new writing booklet.

(a) For what values of x is $\dfrac{x}{x-1} \geq -2$? **3**

(b) The diagram shows the graph of $y = f(x)$.



(i) Sketch a one-third page diagram of $y = f^{-1}(x)$ after making an appropriate restriction on the domain of $f(x)$ so that $y = f^{-1}(x)$ is a one-to-one increasing function. Show intercepts. **1**

(ii) Sketch $y = \dfrac{1}{f^{-1}(x)}$ on the same set of axes, showing all necessary features **2**

Question 11 continues on page 7

Question 11 continued

(c) Let $P(x) = x^3 + 5x^2 + x - 15$.

(i) Show that $P(-3) = 0$. **1**

(ii) Hence, express the polynomial $P(x)$ as $A(x)B(x)$, where $B(x)$ is a quadratic polynomial. **2**

(d) Triangle ABC has vertices $A(-4, 4), B(1, 7)$ and $C(10, 0)$.

(i) Give $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in column vector form. **2**

(ii) Find $\angle BAC$ to the nearest minute and hence find the area of the triangle. **2**

(e) Show that $\int_0^{\frac{\pi}{2}} \sin 7x \sin 3x dx = 0$ **3**

End of Question 11

Please turn over

Question 12 (15 marks) Use a new writing booklet.

(a) Use the principle of mathematical induction to show that for all integers $n \geq 1$ **3**

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2}{4}(n+1)^2 \ .$$

(b) A particle is projected from level ground with an initial velocity of

$5i + 12j$ ms^{-1}. Use g = 9.8 ms^{-2}.

(i) Find the initial speed and angle of projection of the particle. **2**

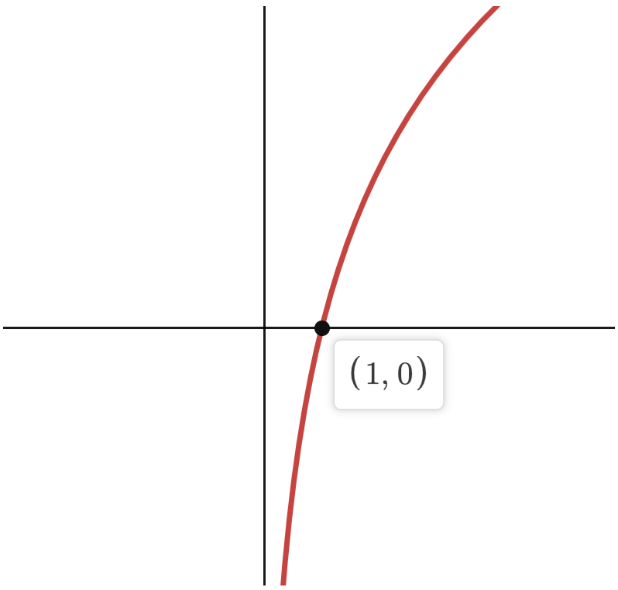
(ii) Show that the displacement vector $\underset{\sim}{d} = 5t\underset{\sim}{i} + (12t - 4.9t^2)\underset{\sim}{j}$ and **3**

hence find the time of flight of the particle to one decimal place.

(iii) Find the horizontal distance travelled by the particle. **1**

(iv) Find the maximum height reached by the particle. **2**

(c) The graph of $y = 4\ln x$ is given below. The part of the graph between $x = 1$ and $x = \mathrm{e}$ is rotated about the y-axis. Find the exact volume of the solid formed. **4**

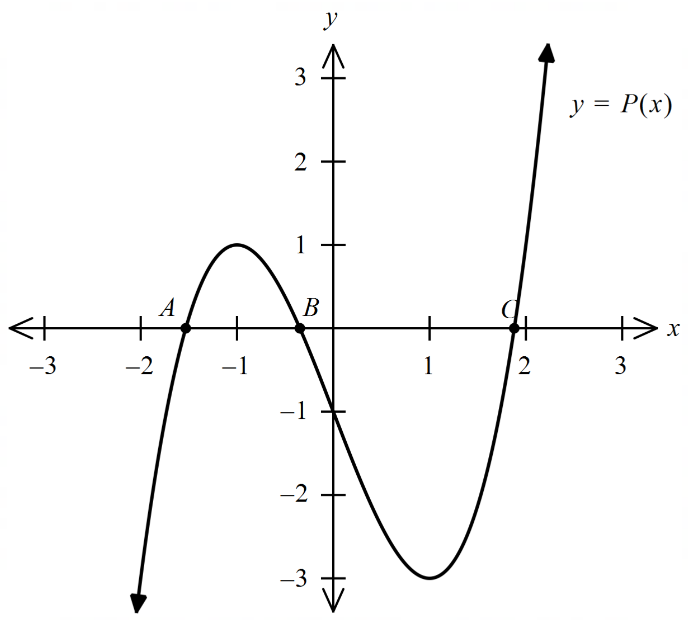


End of Question 12

Question 13 (14 marks) Use a new writing booklet.

(a) (i) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **2**

(ii) The graph of the polynomial $y = P(x) = x^3 - 3x - 1$ is shown below. **4**



Let $x = 2\sin\theta$ and by using part (i), find the roots of $P(x)$: A, B and C, correct to 2 decimal places.

(b) Use the substitution $u = x^3 + 2$ to find the exact value of $\int_0^3 2x^2\sqrt{x^3+2}\,dx$ **2**

(c) Using the substitution $t = \tan\frac{x}{2}$, solve the equation **3**

$2\sin x - 5\cos x - 5 = 0$ for $0 \le x \le \pi$.

(d) Water is poured into a container at a rate of 8 cm^3s^{-1} and the volume of the water in the container is given by **3**

$$V = \frac{3}{2}(h^2 + 8h) \quad \text{where } h \text{ cm is the depth of the water.}$$

Find the rate of change of the depth of the water when $V = 72$ cm^3.

End of Question 13

Question 14 (15 marks) Use a new writing booklet.

(a) Solve the differential equation $\frac{dy}{dx} = \frac{e^x}{2y}$, given the initial condition $y(2) = e$. **3**

(b) A store has reduced the prices on two of their sale items as there had been a number of faults reported by customers.

One of the items is a movie DVD. The store has 3 of these left in stock and it is known that there is a 2.5% probability that each is faulty.

The other item is a music CD with each CD having a 10.8% probability of being faulty. The store has 5 of these left to sell.

A customer chooses to purchase 2 of the movie DVDs and 3 of the music CDs as gifts.

(i) What is the probability that none of these are faulty? **2**

(ii) What is the probability that at least one of the items is faulty? **1**

(c) The equation $y = 7 + \cos\left(\frac{4\pi}{25}t\right) - \sin(\frac{4\pi}{25}t)$ gives the water depth y metres at the wharf, t hours after high tide which occurred at 2 am that morning.

(i) State the amplitude. **1**

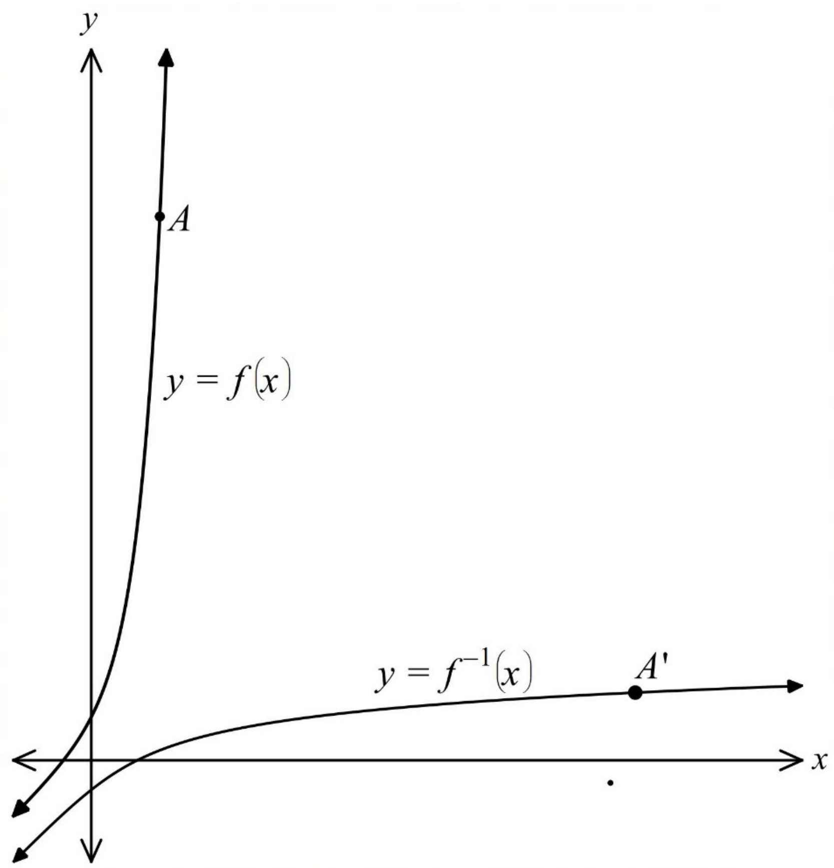
(ii) An overhead power cable obstructs a ship's exit from the wharf. The ship will only be able to exit the wharf if the depth of the water is 8 metres or less. **3**

Find the *latest* possible time before 4 pm that day when the ship can leave the wharf.

Question 14 continues on page 11

Question 14 continued

(d) Consider the graphs of $y = f(x)$ and $y = f^{-1}(x)$ where $f(x) = 1 + x + e^x$ shown below.



The point A lies on $y = f(x)$ and has an x-coordinate of 3. The point A' is the image of point A on $y = f^{-1}(x)$.

(i) Find the exact coordinates of A'. **1**

(ii) It is given that the gradient of the tangent to $y = f(x)$ at A and the gradient to the tangent to $y = f^{-1}(x)$ at A' are reciprocal of each other. Show that the equation of the tangent to $y = f^{-1}(x)$ at A' is $y = \dfrac{x - 1 + 2e^3}{1 + e^3}$ **2**

(iii) Find the coordinates of the point where the tangent to $y = f(x)$ at A meets the tangent from part (ii). **2**

End of Examination

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Section 1

Q1) $\cos 75 = \cos(30+45)$

$= \cos 30 \cos 45 - \sin 30 \sin 45$ (A)

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Q2) $\frac{4 \times 3 + -1 \times x}{\sqrt{4^2+3^2} \times \sqrt{9 \times x^2}} = 0$

$\frac{12-x}{5 \times 3x} = 0$ (D)

$12 - x = 0$

$x = 12$

Q3) $1 - (0.55)^n > 0.9$

$-0.55^n > -0.1$

$0.55^n < 0.1$ (C)

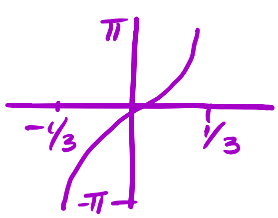
$n \ln 0.55 < \ln 0.1$

$n > \frac{\ln 0.1}{\ln 0.55}$

$n > 3.85$

$n = 4$

Q4)



$-\frac{1}{3} \leq x \leq \frac{1}{3}$ (B)

$-\pi \leq y \leq \pi$

Q5) 15 (C)

Q6) $p(1-p) = \frac{3}{16}$

By trial x Error

$\frac{3}{16} \times \frac{13}{16} \neq \frac{3}{16}$

$\frac{5}{16} \times \frac{11}{16} \neq \frac{3}{16}$ (C)

$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$ ✓

At $t=0$, $x=-2$

$v = \dfrac{1}{t+e}$

Q7) $x = \ln(t+e) + C$

$-2 = \ln e + C$ (D)

$-3 = C$

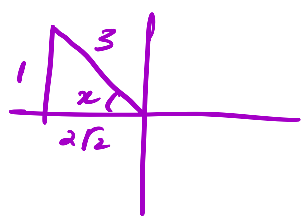
$\therefore x = \ln(t+e) - 3$

At $x=0$, $\ln(t+e) = 3$

$e^3 = t+e$

$e^3 - e = t$

Q8) $\sin x = \frac{1}{3}$



$\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2$

$= \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$ (B)

Q9) B

Q10) $\dfrac{dy}{dx} = \dfrac{-2x}{y-2}$ (A)

when $y=0$ & $x>0$ $\quad \frac{dy}{}>0$

dx

Section 2

Q1) a) $\dfrac{x}{x-1} \geq -2$

$$x(x-1) \geq -2(x-1)^2$$

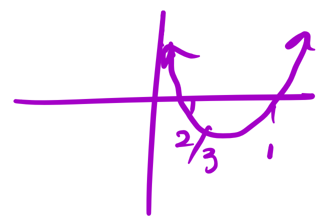
$$x(x-1) + 2(x-1)^2 \geq 0$$

$$(x-1)\left[x + 2(x-1)\right] \geq 0$$ ✓

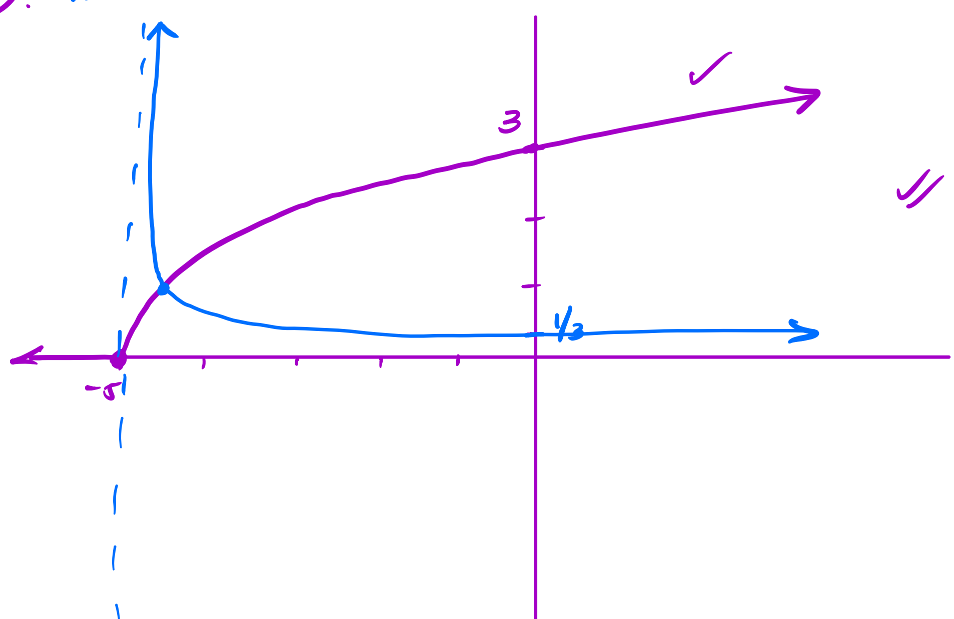
$$(x-1)\left[x + 2x - 2\right] \geq 0$$

$$(x-1)(3x-2) \geq 0$$

$x \leq \frac{2}{3}$ ✓ $\quad x > 1$ ✓



b). (i) & (ii)



✓ ✓✓

c) (i) $P(x) = x^3 + 5x^2 + x - 15$

$P(-3) = (-3)^3 + 5(-3)^2 + (-3) - 15$

$= -27 + 45 - 3 - 15$

$= 0$ ✓

(ii) $(x+3)$ is a factor since $P(-3) = 0$

$$
\begin{array}{r|l}
 & x^2 + 2x - 5 \\
\hline
x+3 & x^3 + 5x^2 + x - 15 \\
 & \underline{x^3 + 3x^2} \\
 & \quad 2x^2 + x \\
 & \quad \underline{2x^2 + 6x} \\
 & \qquad -5x - 15 \\
 & \qquad \underline{-5x - 15} \\
 & \qquad\qquad 0
\end{array}
$$

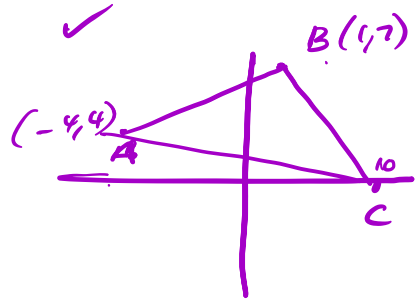
✓✓

$\therefore \; P(x) = (x+3)(x^2 + 2x - 5)$

d). (i) $A = (-4, 4) \quad B = (1, 7) \quad C = (10, 0)$

$\vec{AB} = \begin{pmatrix} 1+4 \\ 7-4 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ ✓

$\vec{AC} = \begin{pmatrix} 10+4 \\ 0-4 \end{pmatrix} = \begin{pmatrix} 14 \\ -4 \end{pmatrix}$



(ii) $\theta = \cos^{-1}\left[\dfrac{5 \times 14 - 3 \times 4}{\sqrt{5^2+3^2} \times \sqrt{14^2+(-4)^2}}\right]$ ✓

$= 46°55'$

$A = \frac{1}{2} \times \sqrt{5^2+3^2} \times \sqrt{14^2+16} \times \sin 46°55'$

$= 31.004 \approx 31u^2$ ✓✓

e). $\int_0^{\pi/2} \sin 7x \sin 3x \, dx$

$= \frac{1}{2}\int_0^{\pi/2} (\cos 4x - \cos 10x)\, dx$ ✓

$= \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 10x}{10}\right]_0^{\pi/2}$ ✓

$= \frac{1}{2}\left[\frac{\sin 2\pi}{4} - \frac{\sin 5\pi}{10} - \frac{\sin 0}{4} + \frac{\sin 0}{10}\right]$

$= \frac{1}{2}\left[0 - 0 - 0 + 0\right] = 0$ ✓

Q12)

a). Prove the result is true for $n=1$

LHS $= 1^3 = 1$

RHS $= \frac{1}{4}(1+1)^2 = \frac{1}{4} \times 4 = 1 =$ LHS

$\therefore$ true for $n=1$

Assume it's true for $n=k$

$1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{k^2}{4}(k+1)^2$

Prove it's true for $n=k+1$

$1^3 + 2^3 + 3^3 + \ldots k^3 + (k+1)^3 = \frac{(k+1)^2}{4}(k+2)^2$

<u>LHS</u>

$$\underbrace{1^3 + 2^3 + 3^3 + \cdots\cdots k^3}_{S(k)} + (k+1)^3$$

$$\frac{k^2}{4}(k+1)^2 + (k+1)^3$$

$$= (k+1)^2\left[\frac{k^2}{4} + k + 1\right]$$

$$= (k+1)^2\left[\frac{k^2 + 4k + 4}{4}\right]$$

$$= (k+1)^2 \frac{(k+2)^2}{4}$$

$$= \frac{(k+1)^2}{4}(k+2)^2$$

$$= RHS$$

$\therefore$ true for $n = k+1$, if the result is true for $n=k$

Then it's true for all $n \geq 1$

b) $\underset{\sim}{v} = 5\underset{\sim}{i} + 12\underset{\sim}{j}$

$$|\underset{\sim}{v}| = \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$\tan\theta = \frac{12}{5}$$

$$\theta = \tan^{-1}\left(\frac{12}{5}\right)$$

$$= 67^\circ 23'$$

(ii) $\underset{\sim}{a} = -g\underset{\sim}{j}$

$\underset{\sim}{v} = -gt\underset{\sim}{j} + C$

At $t=0$, $\underset{\sim}{v} = 5\underset{\sim}{i} + 12\underset{\sim}{j}$

$5\underset{\sim}{i} + 12\underset{\sim}{j} = C$

$\underset{\sim}{v} = -gt\underset{\sim}{j} + 5\underset{\sim}{i} + 12\underset{\sim}{j}$

$\underset{\sim}{v} = 5\underset{\sim}{i} + (12 - gt)\underset{\sim}{j}$

$\underset{\sim}{d} = 5t\underset{\sim}{i} + \left(12t - \frac{gt^2}{2}\right)\underset{\sim}{j} + C$

At $t=0$ $\underset{\sim}{d} = 0\underset{\sim}{i} + 0\underset{\sim}{j}$

$\therefore C = 0$

$\underset{\sim}{d} = 5t\underset{\sim}{i} + \left(12t - \frac{gt^2}{2}\right)\underset{\sim}{j}$

for time of flight, $12t - \frac{gt^2}{2} = 0$

$12t - \frac{9.8t^2}{2} = 0$

$t(12 - 4.9t) = 0$

$t = 0$, $t = \frac{120}{49}$

(iii) At $t = \frac{120}{49}$, $x = 5 \times \frac{120}{49} = \frac{600}{49} = 12.24$

(iv) For max height, $12 - gt = 0$

$12 - 9.8t = 0$

$9.8t = 12$

$t = \frac{12}{9.8}$

At $t = \frac{12}{9.8}$, $y = 12\left(\frac{12}{9.8}\right) - \frac{9.8}{2} \times \left(\frac{12}{9.8}\right)^2$

$= \dfrac{360}{49} = 7.34$

c). $y = 4\ln x$

when $x = 1$, $y = 4\ln 1 = 0$

$x = e$, $y = 4\ln e = 4$

$\dfrac{y}{4} = \ln x$

$x = e^{y/4}$

$V = \pi \int_0^4 \left(e^{y/4}\right)^2 = \pi \int_0^4 e^{y/2}$

(ii) $V = \pi \left[\dfrac{e^{y/2}}{1/2}\right]_0^4$

$= 2\pi \left[e^{y/2}\right]_0^4$

$= 2\pi \left[e^2 - e^0\right] = 2\pi\left[e^2 - 1\right] u^3$

Q13) a) i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\sin(2\theta + \theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$

$= 2\sin\theta(1 - \sin^2\theta) + (1 - 2\sin^2\theta)\sin\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(ii). $p(x) = (2\sin\theta)^3 - 3(2\sin\theta) - 1$

$= 8\sin^3\theta - 6\sin\theta - 1$

$= -2(-4\sin^3\theta + 3\sin\theta) - 1$

from part (a)

$-4\sin^3\theta + 3\sin\theta = \sin 3\theta$.

$\therefore \quad p(x) = -2(\sin 3\theta) - 1$

for roots

$-2(\sin 3\theta) - 1 = 0$

$\sin 3\theta = \frac{-1}{2}$

Base angle $= \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore 3\theta = \frac{-\pi}{6}, -\left(\pi - \frac{\pi}{6}\right), \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ ...

$= \frac{-\pi}{6}, \frac{-5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\theta = \frac{-\pi}{18}, \frac{-5\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}$

$\therefore x = 2\sin\left(\frac{-\pi}{18}\right), 2\sin\left(-\frac{5\pi}{18}\right), 2\sin\frac{7\pi}{18}, 2\sin\frac{11\pi}{18}$ ...

$= -0.35, -1.53, 1.88, 1.88$

from the location of A, B, C from graph given

$x = -0.35, -1.53, 1.88$.

b) $\displaystyle\int_0^3 2x^2\sqrt{x^3+2}\;dx$

$u = x^3+2$

$\dfrac{du}{dx} = 3x^2$

$du = 3x^2 dx$

when $x=0$, $u=2$

$x=3$, $u=29$

$\dfrac{2}{3}\displaystyle\int_0^3 3x^2\sqrt{x^3+2}\;dx$

$\dfrac{2}{3}\displaystyle\int_2^{29}\sqrt{u}\;du$

$\dfrac{2}{3}\left[\dfrac{u^{3/2}}{3/2}\right]_2^{29}$

$\dfrac{2}{3}\times\dfrac{2}{3}\left[29^{3/2} - 2^{3/2}\right]$

$= \dfrac{4}{9}\left(\sqrt{29^3} - \sqrt{2^3}\right)$

$= \dfrac{4}{9}\left(29\sqrt{29} - 2\sqrt{2}\right)$

c). $t = \tan\dfrac{x}{2}$

$2\sin\theta - 5\cos\theta - 5 = 0$

$2\left(\dfrac{2t}{1+t^2}\right) - 5\left(\dfrac{1-t^2}{1+t^2}\right) - 5 = 0$

$4t - 5 + 5t^2 - 5(1+t^2) = 0$

$4t - 5 + 5t^2 - 5 - 5t^2 = 0$

$4t - 10 = 0$

$t = 10/4 = 5/2$ $\quad 0 \leq x \leq \pi$

$\tan x/2 = 5/2 = 2.5 \qquad 0 \le x/2 \le \pi/2$

$x/2 = \tan^{-1}(2.5)$

$x = 2\tan^{-1}(2.5)$

Test $x = \pi$

LHS $2\sin\pi - 5\cos\pi - 5 = 0 = $ RHS

$\therefore$ Solution are $2\tan^{-1}(2.5), \pi$

d). $\frac{dV}{dt} = 8$

$V = \frac{3}{2}(h^2 + 8h)$

$\frac{dV}{dh} = \frac{3}{2}(2h+8)$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{2}{3(2h+8)} \times 8 = \frac{16}{3(2h+8)}$

when $V = 72$

$72 = \frac{3}{2}(h^2 + 8h)$

$\frac{72 \times 2}{3} = h^2 + 8h$

$48 = h^2 + 8h$

$$h^2 + 8h - 48 = 0$$

$$(h-4)(h+12) = 0$$

$$h = 4, -12 \quad \text{since } h > 0, \ h = 4$$

$$\therefore \frac{dh}{dt} = \frac{16}{3(2\times4+8)} = \frac{1}{3}$$

14 a) $\frac{dy}{dx} = \frac{e^x}{2y}$ $\qquad y(2) = e$

$$\int 2y \, dy = \int e^x dx$$

$$\frac{\cancel{2}y^2}{\cancel{2}} = e^x + C$$

$$e^2 = e^2 + C$$

$$C = 0$$

$$\therefore y^2 = e^x$$

$$y = \pm\sqrt{e^x}$$

checking initial condition

$y = \sqrt{e^{x}}$

b) i) $\binom{3}{2}(0.025)^1(0.975)^2 \times \binom{5}{3}(0.108)^2(0.892)^3$

$= 0.59\%$

(ii) $1 - 0.59\% = 99.41\%$

c). $y = 7 + \cos\left(\frac{4\pi}{25}t\right) - \sin\left(\frac{4\pi}{25}t\right)$

$\cos\left(\frac{4\pi}{25}t\right) - \sin\left(\frac{4\pi}{25}\right)t = \sqrt{2}\cos\left(\frac{4\pi}{25}t + \frac{\pi}{4}\right)$

$R = \sqrt{1^2+1^2} = \sqrt{2}$

$\alpha = \tan^{-1} 1/1 = \pi/4$

Amplitude $= \sqrt{2}$

$7 + \cos\left(\frac{4\pi}{25}t\right) - \sin\left(\frac{4\pi}{25}\right)t = 8$

$7 + \sqrt{2}\cos\left(\frac{4\pi}{25}t + \frac{\pi}{4}\right) = 8$

$$\sqrt{2}\cos\left(\frac{4\pi}{25}t+\frac{\pi}{4}\right) = 1$$

$$\cos\left(\frac{4\pi}{25}t+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Base angle $= \pi/4$

$$\frac{4\pi}{25}t+\frac{\pi}{4} = \frac{\pi}{4},\ 2\pi-\frac{\pi}{4},\ 2\pi+\frac{\pi}{4},\ 2\pi+2\pi-\frac{\pi}{4}$$

$$= \frac{\pi}{4},\ \frac{7\pi}{4},\ \frac{9\pi}{4},\ \frac{15\pi}{4}$$

$$\frac{4\pi}{25}t = 0,\ \frac{6\pi}{4},\ \frac{8\pi}{4},\ \frac{14\pi}{4}$$

$$= 0,\ \frac{3\pi}{2},\ 2\pi,\ \frac{7\pi}{2}$$

$$t = 0,\ 9.375,\ 12.5,\ 21.875$$

$t = 12.5$ is the latest before 4pm that day which is 2:30 pm

d) $y = 1+x+e^x$

At $x=3$, $y = 1+3+e^3 = 4+e^3$

$\therefore A'$ is $(4+e^3, 3)$

$y' = 1+e^x$

At $x = 3$

$y' = 1+e^3$

Gradient @ A' $= \dfrac{1}{1+e^3}$

Equation of tangent,

$$y-3 = \frac{1}{1+e^3}\left[x-(4+e^3)\right]$$

$$y-3 = \frac{1}{1+e^3}(x-4-e^3)$$

$$y = \frac{x-4-e^3}{1+e^3}+3$$

$$y = \frac{x-4-e^3+3(1+e^3)}{1+e^3}$$

$$y = \frac{x-4-e^3+3+3e^3}{1+e^3}$$

$$y = \frac{x+2e^3-1}{1+e^3}$$

(iii) The tangents meet on $y=x$

$$\frac{x+2e^3-1}{1+e^3} = x$$

$$x+2e^3-1 = x(1+e^3)$$

$$\cancel{x}+2e^3-1 = \cancel{x}+xe^3$$

$$x = \underline{2e^3-1}$$

$$\overline{\quad e^3 \quad}$$

At $x = \dfrac{2e^3 - 1}{e^3}$

$$y = \frac{\dfrac{2e^3-1}{e^3} + 2e^3 - 1}{1+e^3}$$

$$= \frac{2e^3 - 1 + e^3(2e^3-1)}{e^3(1+e^3)}$$

$$= \frac{(2e^3-1)\cancel{(1+e^3)}}{e^3\cancel{(1+e^3)}} = \frac{2e^3-1}{e^3}$$

$$\therefore \left(\frac{2e^3-1}{e^3}, \frac{2e^3-1}{e^3}\right)$$